MATE WITH BISHOP AND KNIGHT IN KRIEGSPIEL

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§**1. Introduction.** In this paper we deal with the long unsolved problem of whether the king, bishop and knight win against the king alone in the game of kriegspiel. For the purposes of this paper we do not need to give the complete rules of the game. It suffices to say that kriegspiel is the game of chess in which the players are not informed of their opponent's moves. Instead, each player keeps track of the position of his own pieces and tries to gain information about the position of the opponent's pieces as the game progresses. There is a referee who controls the game by keeping track of the true position and by telling the player whose turn it is to move whether a proposed move is a legal chess move or not, using the word "no" to indicate an illegal move. In addition, when a legal move puts a player's king in check, the referee announces this fact to both players and specifies the direction of the check. In our case, this essentially means that he announces whether the check is by the bishop or the knight or both.

Two other rules of chess are not in effect in kriegspiel. The first is the three-position rule. If a position is repeated three times in chess, the player whose turn it is to move may claim a draw. In kriegspiel, a "position" is not only the physical position of the pieces but the information each player has about the opponent's forces. Even if the same physical position occurs three times (as seen by the referee), the information the players have generally changes. This rule is dropped completely in kriegspiel. The second is the

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50-move rule. In chess, if "no significant progress" is made in 50 moves, a player may claim a draw. This rule is also completely dropped in kriegspiel.

The exact rules of the game may differ from one locality to another. In particular, the rules of the game as played in England differ in significant ways from the rules as usually played in this country. For the endgame treated in this paper, the difference in these rules is irrelevant. For the rules as played in England, see the article by Compayne [3]. A complete set of the rules as usually played in this country, the "RAND" rules, Williams [7], may be obtained from the author by request. A collection of kriegspiel problems using the English rules has been published by Anderson [1] under the title *Are There Any?*, a phrase characterizing one of the English rules. In the English rules, a player must ask if there are any pawn tries (possible captures by a pawn) and in the RAND rules, pawn tries are automatically announced. A collection of kriegspiel problems using the RAND rules was presented by L. Shapley [6] at the Ohio State Conference in Game Theory in 1987.
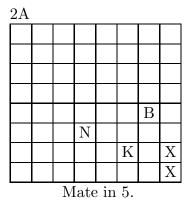
There is a large body of lore with respect to kriegspiel. The elementary mates with queen or rook or two bishops are all known. See Boyce [2] for a description of the mate with the rook. I cannot help mentioning one of the problems of Shapley's remarkable collection, Shapley (1960), that of the rook and king initially placed together in the corner vs a king alone on a semi-infinite board in two directions (the upper right quadrant, say). The rook and king can mate the opposing king with probability one without any information as to the initial whereabouts of the opposing king!

Another of Shapley's remarkable problems, more germane to the subject of this paper, is a position of king, bishop and knight vs. king in which the superior force may mate in 22 moves, Shapley (1973). In this problem knowledge is required of the initial position of the opposing king. However, the solution to this problem plays a role in the solution to the general problem presented here.

In this paper, it is shown that in general (say, with king initially guarding both bishop and knight) the player with bishop and knight can win with probability one. The proof is constructive; a general procedure for winning is explicitly exhibited. In addition, it may be assumed that the player with king alone plays with full information of the past moves of his opponent. The winning procedure involves repeated randomization and so no upper

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bound can be placed on the number of moves required to mate. With king, bishop and knight initially on h8, g8 and h7, respectively, the expected number of moves required by the proposed winning strategy is bounded by 95. This may be compared with the maximum of 34 moves required to mate in the game of chess from a general position, as claimed in Fine [4]. Whether or not there exist a strategy in kriegspiel that guarantees mate within a fixed number of moves is still unknown.

§**2. Notation.** As the general method is rather complex, we shall need a good notation to describe it. We shall refer to the player with king, bishop and knight as white, and as his opponent as black. Squares of the board are denoted in algebraic notation (a1 = lower left corner, h1 = lower right corner, h8 = upper right corner, etc.), and capital letters K, B and N are used to denote king, bishop and knight, respectively. Thus, Kb3 means the king moves to the square b3 (2nd col, 3rd row). On a diagram of the chess board, we indicate the position of the white king, bishop or knight by K, B or N resp., and we indicate our knowledge of the black forces (it is part of the position) by placing an X on all squares which are possible positions for the black king.

We also need a notation that indicates what happens when a "no" is announced by the referee, and the attempted move must be taken back. If a move is allowed, we proceed to the right across the page; if a "no" is announced, the next move tried falls below it in the same column (sometimes several rows below). Position 2A is a simple example of a mate in 5.

2A

Kg3 Bh3 Ne5 Nf3 Bg2 mate
Ne5 Kg3 Bh3 Bg2 Nf3 mate

(If N at e4, replace Ne5 by Nd2.)

Mate in 5.

This indicates that white first tries the move Kg3. If it is allowed, he continues Bh3, Ne5, Nf3, Bg2 mate. If it is a "no", he plays instead Ne5, Kg3, etc., in either case mating

on the fifth move. The remark in parenthesis implies there is a mate in 5 if the knight is originally on e4, and that mate may be achieved by replacing the move Ne5 where it occurs by the move Nd2 (twice).

A choice of move may depend on whether a check was announced at the previous move. If the choice does so depend, the move when no check is announced follows it to the right, and the move when check is announced follows the "if+" symbol that falls below the move on which the check was made. If the choice does not depend on the announcement of check, the "if+" symbol will be omitted, indicating that the announcement may be ignored. Consider Position 2B.

White first tries Kf2. If that succeeds, he has mate in 5 more moves using the previous position, 2A. If Kf2 produces a "no" from the referee, he plays Bh3 and listens for check. With no check, he first tries Kg3; if this is successful, he plays Ne5, Bg2, Nf3 mate; otherwise, he plays Nf2 etc. and mates on the seventh move. If Bh3 gave check, he plays Kg3 etc. mating on move 5.

2B

					B		
		N		K			
							X
				X	X	X	

Mate in 7.

```
Kf2  2A
Bh3  Kg3  Ne5  Bg2  Nf3   mate
     Nf2  Ng4  Kg3  Ne5  Bg2  Nf3   mate
if+  Kg3  Ne5  Nf3  Bg2   mate
```

Shapley's 1973 problem of mate with bishop and knight in 22 moves is given below. The first two moves constrict the black king to an area consisting of 6 squares, from which he cannot escape. The next five moves bring the white king into the action at e6. This reaches a position that is a mirror image of position 3A, treated in the next section, which is a mate in 15.

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Shapley (1973)

					X		
				N			
	B		K				

Mate in 22.

Ne5 Bg5 Ke2 Ke3 Ke4 Kd5 Ke6 3A

The general method of cornering and mating the black king may now be described. In order to mate with probability one, white can never leave the bishop or the knight in a position of possible capture by black's king. The general method requires that the knight and bishop be set up so that they cannot be attacked while the white king sweeps the board in search of the black king. The position to aim for in order to initiate the sweep is 2C, in which it is known that the black king is not in the upper right corner. (This must be checked first.) That the sweep can be implemented successfully is proved in sections 4 to 6.

2C

X	X	X					
X	X		X				
X	X	X	X	N	K	B	
X	X		X				
X	X	X				X	X
X	X	X		X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

The sweep of the white king in search of the black then takes the following path: Ke7 (or Ke5), Kd6, Kc6, Kb7, Kc6, Kb5, Kb4, Kc3 (not Kb3 which might stalemate the black king at a1), Kb2, Kc2, Kd2, Ke2, Kf2, Kg2. At this point, with the black king on g4 or h4, white maneuvers to confine black to the lower right corner while bringing the bishop to the diagonal d1–h5, and completing the mate as in Shapley's problem. This maneuver seems to require randomization, as does the strategem in the general solution of moving the king from c6 to b7 to c6 to b5.

In the next section, we exhibit the mates when the black king is confined to the lower right corner below the diagonal d1–h5, occupied by the bishop. This includes the solution to Shapley's problem. In section 4, we examine the procedure of bringing about the positions of section 3 when the black king is confined to the lower right corner below the long diagonal b1–h7 occupied by the bishop. In section 5, we examine the sweep from c6 to b2, and in section 6, the sweep from position 2C to c6. In the final section, we discuss setting up the initial position, 2C.

§**3. The small diagonal.** In this section, we consider the simple mates that occur when the bishop is on the small diagonal, d1–h5, and the black king is trapped in the lower right corner. The first position is essentially the Shapley problem after the first 7 moves.

3A

Mate in 15.

Ke2	3B		
Kd2	Ke2	3B	
Nc5	Nd3	Kf3	2B
	Kf4	Kf3	2B

3B

Mate in 13.

Kf2	2A			
Kf3	Kf2	2A		
	Nc5	Ke2	Kf2	Ne4 2A
		Ne4	Kf2	2A
			3C	
	Ke3	Nd3	Kf3	2B
		Kf4	Kf3	2B
Ke1	Kf2	2A		
	3C			

3C

Mate in 8.

Bh3 Bf1 Kf2 3D
          Ke1 Kf2 3D

3D: Kg3 Nd2 Bg2 Nf3 mate
     Nf6 Bg7 Ng4 mate

(If K at e1, replace Ke1 by Ke2.)

Another important class of mates occurs with the knight on d3 instead of e4.

3E

Mate in 15.

Kf4 Kg4 3F
     Kg5 Kg4 3F
          Bg4 Kf4 3G
     Bf3 Kg5 Kg4 Kg3 3H
               Bg4 Kf4 3G
Kg5 Kg4 3F
     Bf3 Kg4 Kg3 3H
          Bg4 Kf4 3G

3F

Mate in 12.

Kg3 3H
Bf3 Kg3 3H
     Kh4 Kg3 3H

3H: Bg4 Kf3 2B

3G: Kf3 2B
     Bf5 Kf3 Bg4 2B

3I

Mate in 15.

Kg3 3H
Kg4 Kg3 3H
Kg5 Kg4 Kg3 3H
     Bg4 Kf4 3G
Be2 Kg5 Kg4 3F
          Bg4 Kf4 3G

Finally, a position with the knight on e6. The procedure is to transform to 3A.

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3J

			N				
							X
			K	B	X	X
							X
			X	X	X	

Mate in 24.

Kf2   Ke3   Kf4   Nc5   Ne4   3K
                  Nc5   Ne4   3K
Nc5   Ne4   3K

3K

			N			X
			K	B		X
						X
			X	X	X	

Mate in 19.

Kf2   Be2   Ke3   Kf4   3A
Nd6   Nf5   3I

§**4. The big diagonal.** The critical position in moving from the big diagonal to the small diagonal is 4A. It seems to require randomization. It is known that black has just moved his king from g4 to either h4 or h3. If white knew which he would have a mate in a bounded number of moves. If black has moved to h3, white has mate in 18 moves beginning with Bh5. If black has moved to h4, white has a mate in 25 moves, beginning with Kf3, Ng5+. Fortunately, white has a method of testing to find which square black has moved to. He may test either Kg2 immediately, or triangularize with Kf3, Kg2. If he gets a "no", he may proceed to mate as described; otherwise, he will be back where he started in 2 moves if he tried Kg2 immediately, or 3 moves if he tried Kf3, Kg2. If white is only interested in winning with probability one (w.p. 1), he may choose randomly between these two possibilities w.p. 1/2 each, and repeat the choice independently until he gets a "no". If he wants to win as quickly as possible, he might randomly choose between the possibilities w.p. θ and $1 - \theta$, and select θ to minimize the maximum expected number of moves to mate.

4A

w.p. θ: Kg2   Kf2   4A
                     Bh5   Bf3   Ng5   Ne4   4C

w.p. $1 - \theta$:   Kf3   Kg2   Kf2   4A
                      4B

EM < 26.

With this understanding, position 4A becomes a recursive game, Γ, with the following structure:

$$\Gamma = \text{black} \begin{array}{c} \\ \text{Kh3} \\ \text{Kh4} \end{array} \overset{\displaystyle \begin{array}{cc} \text{white} \\ \text{Kg2} \quad\quad \text{Kf3} \end{array}}{\begin{pmatrix} 18 & \Gamma + 3 \\ \Gamma + 2 & 25 \end{pmatrix}}$$

This game may be solved by the usual method of letting $V = \mathrm{val}(\Gamma)$, and equating V to the value of the game on the right side above with Γ replaced by V. (See the book of Owen [5] for an elementary treatment of such problems.) This equation is $V(2V - 38) =$

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$(V + 2)(V + 3) − 450$, which gives $V = (43 + \sqrt{73})/2 = 25.772\cdots$, and white's optimal strategy is to choose Kg2 with probability $\theta = (13 − \sqrt{73})/16 = .2785\cdots$ and Kf3 with probability $1 − \theta = .7215\cdots$. This strategy of white guarantees that the expected number of moves until mate is bounded by $V < 26$ moves. We denote by EM the expected number of moves required to mate.

4B

		N		B			
			K		X		
					X		

Mate in 24.

Ng5 Bd3 Kf2 Kg3 Nh3 Be4 mate
              Bf1 Bg2 Nf3 mate
              Kg3 Be2 Nh3 Bf3 mate
if+ Ne4 Kg3 Bh5 Bg4 Kf4 Ke3 3A
          Bf7 Kf4 Bh5 Kg3 Bg4 Kf4 Ke3 3A
                      Be2 3A

4C

		N					
			B		X		
			K				

Mate in 14.

Ke3 Kf4 Nc5 Nd3 4D
              if+ Kg3 Bg4 Kf3 2B

4D

			K				
		N		B		X	
					X		

Mate in 10.

Kg3 Bg4 Be2 Nf4 Nh3 Bf3 mate
Ne1 Kg4 Kg3 Nd3 Be2 Nf4 Nh3 Bf3 mate
         Ng2 Be4 Kg4 Kg3 Kf2 Bf5 Ne3 Nf1 Be4 mate
                     Bd3 Nf4 Nh3 Be4 mate
                     if+ Nf4 Be2 Nh3 Bf3 mate

Once the black king is trapped in the lower right corner below the large diagonal as in 4E, the method of bringing about 4A is fairly simple:

4E

		N		B	
				X	X
K		X	X	X	X
		X	X	X	X
X	X	X	X	X	X

6 moves to 4F.
EM < 42.

Kc2 4F
Kb2 Kc2 4F
　　　Bc2 Kc1 Bg6 Kc2 4F
Bf5 Kb2 Kc2 Bg6 4F
　　　　　Bc2 Kc1 Bg6 Kc2 4F

(One more move is required
if the king starts on b2 or b3.)

4F

		N		B	
				X	X
		X	X	X	X
K		X	X	X	X
		X	X	X	X

5 moves to 4G.
EM < 36.

Kd2 4G
Bd3 Kd2 Bg6 4G
　　　Bg6 Kd2 4G
　　　　　Kc1 Kd2 4G
　　　　　　　Kd1 Kd2 4G
　　　　　　　Bh5 Kd2 Ke3 Bf3 3J
　　　　　　　　　　Be2 Ke3 Bf3 3J

4G

		N		B	
				X	X
			X	X	X
	K		X	X	X
			X	X	X

2 moves to 4H.
EM < 31.

Ke2 4H
Kd3 Ke2 4H
　　　Bh5 Ke3 Bf3 3J
　　　　　Kd2 Ke3 Bf3 3J

4H

		N		B	
				X	X
				X	X
		K		X	X
				X	X

3 moves to 4A.
EM < 29.

Kf2 Kg2 Kf3 4B
　　　　　Kf2 4A
　　　Bh5 Bf3 3J   (after one move; mate in 23)
Bh5 Bf3 Ke3 3J

§**5. The sweep from c6 to c3.** Once position 5A has been obtained, white must sweep down the left side of the board.

5A

X		K		N		B	
X							
X	X	X				X	X
X	X	X		X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

2 moves to 5B.
EM < 72.

```
Kb5  5C
Be4  Kb6  Kb5  Bg6  5C
          Kc5  Kb5  Bg6  5C
                    Bc6  3I
          Ka5  Kb5  Bg6  5C
          5B
     Kc5  Kb5  Bc6  Kc5  3I
          Bc6  3I
          if+  Kb5  Be8  Bg6  5C
          3I
```

If it were black's turn to move in 5B, white would have an easy three move sequence to arrive at 5C, detailed in 5B′ below. So white must lose a move, say by triangularization of the bishop (Bf5 Bg6 Be4), while keeping black out of d5 (by Kc6 Kb6). This takes 5 moves.

5B

	K			N			
	X			B			

5 moves to 5B′.
EM < 70.

```
Bf5  Kb5  Bg6  5C
     Kc5  Kb5  Bg6  5C
               Bc2  Ba4  Bc6  3I
     Kc6  Kb5  Bg6  5C
          Bg6  Kb5  5C
               Kc5  Kb5  5C
                    Be8  Bc6  3I
                    if+  Kb5  Bg6  5C
                         Bc6  3I
          Be4  Kb5  Bg6  5C
               Kb6  5B′
```

From position 5B′, which is position 5B with black to move, white reaches 5C as follows:

```
5B′:          Kb5  Bg6  5C
              Ka5  Kb5  Bg6  5C
3moves to 5C. Kc5  Kb5  Bg6  5C
  EM < 65.    Bc6  3I
```

White can save about a move and a half by randomizing in 5B between Ka6 and Kc6.

The resulting recursive game to reach 5C is

$$
\Gamma = \text{black} \quad
\begin{array}{c}
\\
\text{Ka5} \\
\text{Kc4}
\end{array}
\begin{array}{c}
\text{white} \\
\begin{array}{cc}
\text{Ka6} \quad\quad \text{Kc6}
\end{array} \\
\begin{pmatrix}
\Gamma + 2 & 5 \\
4 & \Gamma + 2
\end{pmatrix}
\end{array}
$$

which has value $(9 + \sqrt{17})/2 = 6.56\cdots$ moves to reach 5C instead of 8. We do not include this strategem in our proposed strategy for two reasons, the first of which is to emphasize that in this spot randomization can be avoided, unlike what I believe to be true for 4A and 6D. Second, the randomization suggested for 5B differs from the other two randomizations in that it requires black to be blind. We prefer to give a strategy that works even if black sees white's moves. (Black may serve as referee!) The rest of the sweep is easy.

5C

Kb4 5E
Kc5 Kb4 5E
       Kc4 5D
       Nd4 Kb4 Ne6 5E
              Kc4 Kb4 Ne6 5E
                     Kb5 Kb4 Ne6 5E
                     Bf7 Kc3 Nf5 4E
              Bh7 Kb4 Ne6 Bg6 5E

6 moves to 5E.
EM < 62.

5D

Kb4 5E
Kb3 Be8 Bd7 Kc4 3E
Nc5 Kb4 Ne6 5E
if+ Kb5 Kb4 Ne6 5E
       Be8 Ne6 Bd7 3E

4 moves to 5E.
EM < 60.

5E

			N		B		
	K					X	X
				X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

4 moves to 5F.
EM < 56.

Kc3 Kb2 4E
    Nc5 Kb2 Ne6 4E
        Nb3 Bd3 5F
Kb3 4E
Nc5 Kc3 Kb2 Ne6 4E
        Nb3 Bd3 5F

5F

	B						
	K						
	N						
X	X	X					

10 moves to 4E.
EM < 52.

Kd2 Bd5 Kc3 Ba2 5G
Kb2 Bb5 Kc3 Be2 Bc4 Ba2 5G
Ba2 5G

5G: Nd4 Ne6 Bb1 Bg6 4E

§**6. The sweep from f6 to c6.** We initiate the sweep from 6A with the aim of reaching

6C and then 6D. Here is how this part is done.

6A = 2C

X	X	X					
X	X		X				
X	X	X	X	N	K	B	
X	X		X				
X	X	X				X	X
X	X	X		X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

5 moves to 6C.
EM < 86.

Ke5 Kd6 6C
    Be4 6B
    if+ Kd6 Bg6 6C
        6B
Ke7 Kd6 6C
    Bh7 Kd6 Bg6 6C
        Kf6 Ke5 Kd6 6C
                Be4 6B
            Be4 Ke5 6B
Be4 Ke5 6B

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		X		X			
			X	X	X		
				N			
				K			
				B			

10 moves to 5C.
EM < 72.

Nf4 Bd5 Bb3 Nd3 Kf4 Kg4 5C
                    Bd1 Bf3 3I
                    if+ Kg4 Bb3 5C
                                Bf3 3I
                Bd1 Kf4 Kg4 Bb3 5C
                                Bf3 3I
            if+ Kf4 Bf3 3I
                Kf5 Be2 3E
                Nf4 Kf5 Kg4 Nd3 Bb3 5C
                    Nd3 Kf4 Bf3 3I

X	X	X					
X	X						
X	X		K	N		B	
X	X						
X	X	X				X	X
X	X	X		X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

1 move to 6D.
EM < 81.

Kc6 6D
Kc7 Be4 Kb6 Kc6 Bg6 5A
          Kc6 Kb5 Bc6 Kc5 3I
              5A′ (after the move Be4)
          Bd3 Kc6 Bg6 5A
Be8 Kc5 Bc6 3I
     Bc6 Kc5 3I

The difficult part is progressing from 6D to 5A. Again we use a randomization.

X	X	X					
X							
X		K		N		B	
X							
X	X	X				X	X
X	X	X		X	X	X	X
X	X		X	X	X	X	X
X		X	X	X	X	X	X

EM < 80.

w.p. θ: Kb7 Kc6 5A
                            6E
                    Be8 Kc5 Bc6 3I

w.p. $1 - \theta$:   Kb5 Kc6 6D
                            6F
                    5A

In positions 6E and 6F one must admit failure and regroup to start all over again. First of all, the knight may be attacked; so it must move. Fortunately, there is a simple retreat that will safeguard both bishop and knight, namely Ng5, Nh7. Now it is a matter of bringing the king to f6 and then moving the knight back into position to reach 6A. Let us estimate the maximum number of moves that are required for this. After the first two knight moves, when white moves, black should have the opposition, and should be able to

make white waste 4 more moves, for a total of 12 moves. A typical struggle from 6E is as follows:

```
6E:  Ng5  Nh7  Kc6
               Kc7
               Kc8
          Bh5  Kc8  Kd8
                    Kc7  Kd8  Ke8  Kf7
                                   Bg6  Kf7  Kf6
                                             Nf8  Kf6  Ne6  6A
```
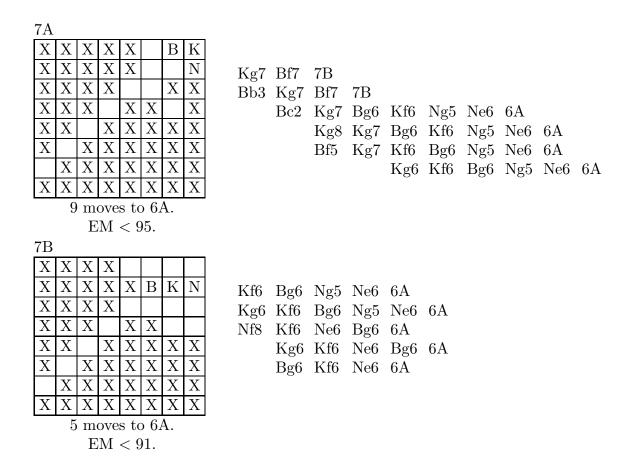
From 6A, it takes 6 more moves to return to 6D for a total of 19 moves, counting the one at the start. From 6F, the procedure is simpler; one can attain a reversed 6A position in just 6 moves, making 13 moves to return to 6D.

6F: Nc5 Kc4 Bc2 Nd3 Kc3 Bb3 6A

We are now able to evaluate the expected number of moves required to proceed from 6D to 5A. We consider three possible distinct strategies for black when white first reaches c6; he can hope that white will try b7, and put himself at c4 to attack the knight; or he can hope that white will try b5 and either leave himself out of the action, say at b3, or hide at c8, say, to attack the knight. This leads to the following recursive game:

$$
\Gamma = \text{black} \quad
\begin{array}{c}
 \\
\text{Kc4} \\
\text{Kb3} \\
\text{Kc8}
\end{array}
\begin{array}{c}
\text{white} \\
\begin{array}{cc}
\text{Kb5} & \text{Kb7}
\end{array} \\
\left(
\begin{array}{cc}
72 & \Gamma + 19 \\
\Gamma + 2 & 74 \\
\Gamma + 13 & 18
\end{array}
\right)
\end{array}
$$

Solving as before, we find the top two rows active, and solve for the value as the root of the equation, $V(2V - 125) = (V + 19)(V + 2) - 5328$, namely $V = 73 + \sqrt{39} = 79.245 \cdots$. The optimal probability for white is $\theta = .276 \cdots$.

§**7. Getting started.** Now, it is a matter of showing how the initial setup 6A can be obtained. We illustrate how this is done from the starting position in which the white forces are squeezed into the upper right corner, with king at h8, bishop at g8 and knight at h7. It takes a total of 9 moves.

7A

X	X	X	X	X		B	K
X	X	X	X	X			N
X	X	X	X			X	X
X	X	X		X	X		X
X	X		X	X	X	X	X
X		X	X	X	X	X	X
	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X

9 moves to 6A.
EM < 95.

Kg7 Bf7 7B
Bb3 Kg7 Bf7 7B
    Bc2 Kg7 Bg6 Kf6 Ng5 Ne6 6A
        Kg8 Kg7 Bg6 Kf6 Ng5 Ne6 6A
        Bf5 Kg7 Kf6 Bg6 Ng5 Ne6 6A
            Kg6 Kf6 Bg6 Ng5 Ne6 6A

7B

X	X	X	X				
X	X	X	X	X	B	K	N
X	X	X	X				
X	X	X		X	X		
X	X		X	X	X	X	X
X		X	X	X	X	X	X
	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X

5 moves to 6A.
EM < 91.

Kf6 Bg6 Ng5 Ne6 6A
Kg6 Kf6 Bg6 Ng5 Ne6 6A
Nf8 Kf6 Ne6 Bg6 6A
    Kg6 Kf6 Ne6 Bg6 6A
    Bg6 Kf6 Ne6 6A

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