Solutions to the Exercises of Section 1.5.

1.5.1. If $\delta = (.1, .5, .1, .3) \in D^*$ is used, then the probability that a_1 is chosen if x_1 is observed is $\pi_1 = .1 + .5 = .6$; the probability that a_1 is chosen if x_2 is observed is $\pi_2 = .1 + .1 = .2$. So δ is equivalent to $(.6, .2) \in \mathcal{D}$.

1.5.2. If $(\pi_1, \pi_2) = (1/3, 3/4) \in \mathcal{D}$ is equivalent to $(p_1, p_2, p_3, p_4) \in D^*$, then $p_1 + p_2 = \pi_1 = 1/3$ and $p_1 + p_3 = \pi_2 = 3/4$. There are many solutions to these equations that satisfy $p_1 + p_2 + p_3 + p_4 = 1$ with all p_j nonnegative, and all are equivalent to (π_1, π_2). Fix p_1 and solve: $p_2 = 1/3 - p_1, p_3 = 3/4 - p_1, p_4 = 1 - p_1 - p_2 - p_3 = p_1 - 1/12$. All p_j are nonnegative provided $1/12 \le p_1 \le 1/3$. Hence, any point $(p_1, 1/3 - p_1, 3/4 - p_1, p_1 - 1/12)$ with $1/12 \le p_1 \le 1/3$ is equivalent to $(1/3, 3/4) \in \mathcal{D}$.

1.5.3. $D = \{d : \mathcal{X} \to \mathcal{A}\}$ has m^n elements. So D^* is $m^n - 1$ dimensional (minus 1 because the p_j must add to 1). $\mathcal{A}^*$ is $m - 1$ dimensional, so $\mathcal{D}$ is $n(m-1)$ dimensional. For example, if $m = 10$ and $n = 5$, then D^* is 99,999 dimensional and $\mathcal{D}$ is 45 dimensional.