Large Sample Theory

Ferguson

Exercises, Section 15, Asymptotic Joint Distribution of Extrema.

1. Find the asymptotic distribution of the range when sampling from a Cauchy distribution, $\mathcal{C}(0,1)$.

2. Find the asymptotic joint distribution of the range, $R_n = X_{(n:n)} - X_{(n:1)}$, and of the midrange, $M_n = (X_{(n:n)} + X_{(n:1)})/2$, when sampling from a Pareto distribution with density $f(x) = 1/x^2$ for $x > 1$.

3. (a) Find the asymptotic distribution of the ratio of the smallest to the next smallest observation, when sampling from a uniform distribution, $\mathcal{U}(0,1)$.

(b) What is the distribution of this ratio for a fixed sample size?

4. Find the asymptotic distribution of the range when sampling from a distribution with density $f(x) = (1 + \theta(2x - 1))$ for $0 \leq x \leq 1$, where $0 < \theta < 1$ is a known parameter.