**Large Sample Theory**
Ferguson

**Exercises, Section 10, Asymptotic Power of the Pearson's Chi-Square Test.**

1. An important method used by some gamblers to change the odds in dice games is to shave two opposite sides of a die slightly so as to increase the probability that those two sides come up. It doesn't take much shaving to change the odds of the casino game "craps" to be in the roller's advantage.

(a) Suppose you roll a die 150 times to test the hypothesis of fairness at the 5% level of significance. What is the approximate power at the alternative, $p_3 = p_4 = .20$, and $p_1 = p_2 = p_5 = p_6 = .15$? If you don't want to interpolate in the Fix Tables, use the noncentral $\chi^2$ tables in the Statistical Tables at
`http://www.math.ucla.edu/~tom/distributions/CONTENTS.html`.

(b) How large a sample size would you need for a test of the fairness of a die at the 5% level of significance to have power of .9 at this alternative?

2. Suppose that $X_1, \ldots, X_n$ is an independent identically distributed sequence of $d$-dimensional vectors from a distribution with unknown mean $\mu$ and nonsingular covariance matrix $\Sigma$. According to Lemma 9.2, Hotelling's $T^2$ is asymptotically $\chi^2_d$. Suppose we use

$$T_n^2 = n(\overline{X}_n - \mu_n)^T S_n^{-1}(\overline{X}_n - \mu_n)$$

where $\mu_n$ is a sequence of vectors converging to the true $\mu$. Formulate and prove a theorem on the limiting non-central $\chi^2_d$ distribution for $T_n^2$.

3. Suppose a sample of size 60 is taken from the hatchings of a litter of lady bird beetles, and the offspring are divided into the two-by-two contingency table using the dichotomies, male/female and spotted/plain. The data are: 15 spotted-male, 21 spotted-female, 17 plain-male, and 7 plain-female.

(a) Find Pearson's $\chi^2$ for testing the hypothesis that all 4 cells have equal probability, 1/4. How many degrees of freedom does the $\chi^2$ have?

(b) Find the noncentrality parameter for the alternative that specifies P(spotted-male)=.20, P(spotted-female)=.35, P(plain-male)=.30, and P(plain-female)=.15.

(c)Find the sample size needed to get power .9 at this alternative when testing at the 5% level of significance.